2014

MATHEMATICS

(Major)

Paper : 1.2

(Calculus)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : $1\times10=10$

(a) Write down the *n*th derivative of $\cos(3x+5)$.

(b) If $u=f\left(\frac{y}{x}\right)$, then obtain the value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$.

(c) The equation of a curve is $\log y = x\log a$. What is the length of the subtangent to the curve at the point $P(x, y)$?

(d) Define the curvature of a curve at point on it.

(e) Write down the equation of the asymptote of the curve $xy-3x-4y=0$ which is parallel to the x-axis.

(f) If $f(x, y)=\log(x^2+y^2)$, then determine $\frac{\partial f}{\partial y}$.

(g) Choose the correct answer :

$\int\sqrt{a^2-x^2}\,dx$ (ignoring the constant of integration) equals

(i) $\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\cos^{-1}\frac{x}{a}$

(ii) $\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}$

(iii) $\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\cosh^{-1}\left(\frac{x}{a}\right)$

(iv) $\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right)$

(h) Write down the value of $\int_{-a}^{a} x^3 f(x)\,dx\,(a \neq 0)$ where f is an even function.

(i) Evaluate $\int_{-\pi/2}^{\pi/2}\cos x\,dx$.

(j) Write down the intrinsic equation of the catenary $y=c\cosh\left(\frac{x}{c}\right)$.

2. Answer the following questions : 2×5=10

(a) If $y=\log(ax+x^2)$, then find y_n.

(b) Find $\frac{ds}{d\theta}$ for the curve $r=ae^{\theta\cot\alpha}$.

(c) Show that

$$\int_0^{\pi} xf(\sin x)\,dx=\frac{\pi}{2}\int_0^{\pi} f(\sin x)\,dx$$

(d) Show that the area of a loop of the curve $r=a\cos 2\theta$ is $\frac{\pi a^2}{8}$.

(e) Find the volume generated by revolving about OX, the area bounded by $y=x^3$ between $x=0$ and $x=2$.

3. Answer the following questions : 5×4=20

(a) If $u=F(y-z, z-x, x-y)$, then prove that

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$$

Or

If $y=f(x+ct)+\phi(x-ct)$, then show that

$$\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}$$

(b) Trace the curve $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$; $-\pi \le \theta \le \pi$.

Or

Prove that the sum of the intercepts of the tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ upon the coordinate axes is constant.

(c) Evaluate :

$$\int \frac{dx}{3 + 5\cos x}$$

Or

$$\int \frac{dx}{(1+x)\sqrt{1 + 2x - x^2}}$$

(d) Find the perimeter of the circle $x^2 + y^2 = a^2$.

4. Answer *either* (a) *or* (b) :

(a) State the Leibnitz theorem and use it to prove the following : 2+5+3=10

(i) $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$ where $y = (\sin^{-1} x)^2$

(ii) $y_n = \frac{(n-1)!}{x}$ if $y = x^{n-1}\log x$

(b) Define homogenous function and its degree. Also answer the following : 2+4+4=10

(i) If u is a homogeneous function in x and y of degree n having continuous partial derivatives, then prove that

$$\left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}\right)^2 u = n(n-1)u$$

(ii) If $u = \cos^{-1}\frac{x+y}{\sqrt{x} + \sqrt{y}}$, then show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$$

5. Answer *either* (a) *or* (b) :

(a) (i) Find the asymptotes of the curve $x^4 - x^2y^2 + x^2 + y^2 - a^2 = 0$.

(ii) If ρ_1 and ρ_2 are the radii of curvature at the ends P and D of conjugate diameters of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

show that

$$\rho_1^{2/3} + \rho_2^{2/3} = \frac{a^2 + b^2}{(ab)^{2/3}}$$

5+5=10

(b) Define cusp and isolated points. Search for double points on the curve

$$x^2y + x^3y + 5x^4 = y^2$$ 2+8=10

6. (a) If $u_n = \int_0^{\pi/2} \theta \sin^n \theta \, d\theta$ and $n > 1$, then prove that

$$u_n = \frac{n-1}{n} u_{n-2} + \frac{1}{n^2}$$ 5

(b) If $I_n = \int (a^2 + x^2)^{n/2} \, dx$, then show that

$$I_n = \frac{x(a^2 + x^2)^{n/2}}{n+1} + \frac{na^2}{n+1} I_{n-2}$$ 5

7. (a) Find the area of the region bounded by the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$. 5

(b) Find the surface area of the solid generated by revolving the cardioid $r = a(1 - \cos\theta)$ about the initial line. 5

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